Gamma Guidance Schemes for Flight in a Windshear

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This paper is concerned with guidance strategies for near-optimum performance in a windshear. The takeoff problem is considered with reference to flight in a vertical plane. In addition to the horizontal shear, the presence of a downdraft is assumed. A gamma guidance scheme, based on the absolute path inclination, is presented. This approach needs local information on the windshear and the downdraft. The gamma guidance scheme produces trajectories that preserve the basic properties of the optimal trajectories. The relation between the gamma guidance scheme and the acceleration guidance scheme is explored. In logic, these two guidance schemes are complementary to one another; in implementation, they yield almost identical results. Although local information on the windshear and the downdraft will be available in future aircraft, it might not be available on current aircraft. Hence, we present a simplified gamma guidance scheme (quick transition to horizontal flight) that is useful for flight in severe windshears. The simplified gamma guidance scheme yields trajectories that are close to the optimal trajectories in severe windshears; in addition, it is easy to implement as a practical piloting technique.

Nomenclature

D = drag force, lb
g = acceleration of gravity, ft s^{-2}
h = altitude, ft
K = gain coefficient
L = lift force, lb
m = mass, lb ft^{-1} s^2
T = thrust force, lb
V = relative velocity, ft s^{-1}
V_e = absolute velocity, ft s^{-1}
W = mg = weight, lb
W_h = h-component of wind velocity, ft s^{-1}
W_x = x-component of wind velocity, ft s^{-1}
x = horizontal distance, ft
α = relative angle of attack, rad
α_e = absolute angle of attack, rad
β = engine power setting
γ = relative path inclination, rad
γ_e = absolute path inclination, rad
δ = thrust inclination, rad
θ = pitch attitude angle, rad
OT = optimal trajectory
GGT = gamma guidance trajectory
SGGT = simplified gamma guidance trajectory
CPT = constant pitch trajectory
MAAT = maximum angle of attack trajectory

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I. Introduction

At the 1986 ICAS congress, Mulally and Higgins discussed the major causes of atmospheric disturbances affecting the safety of flight.[1] They identified three main threats to aircraft safety: high-altitude turbulence, winter conditions, and low-altitude windshear. They felt that, because of improvements in onboard weather radar, ground-based weather radar, control system design, instrumentation design, flight crew training, and ground crew training, high-altitude turbulence upsets have been nearly eliminated and winter operations incidents have been greatly reduced. They also felt that, with high-altitude turbulence and winter operations essentially under control, the windshear problem has become of increasing relative importance.[2] Indeed, two recent aircraft accidents, involving considerable loss of life, have focused the attention of the engineering and scientific community on the windshear problem: one accident occurred at New Orleans International Airport (Pan Am Flight 759 of July 9, 1982) and involved a Boeing B-727 in takeoff;[3] the other accident occurred at Dallas-Fort Worth International Airport (Delta Airlines Flight 191 of August 2, 1985) and involved a Lockheed L-1011 in landing.[4,5] Therefore, the development of effective guidance schemes and piloting strategies is a pressing task for flight safety.

Some control schemes for flight under windshear conditions were investigated in Refs. 6 and 7 with the purpose of recovering the nominal trajectory occurring in the absence of windshear. A major difficulty occurs in strong and even moderate windshears: because both the angle of attack and the power setting are bounded, the recovery of the nominal trajectory might not be feasible. This makes it imperative to study optimal trajectories under windshear conditions.[8-10]

Optimal trajectories were investigated in Refs. 9 and 10 under the assumption that global information on the wind flowfield is known in advance. With particular reference to takeoff, it was concluded that: (P1) for weak-to-moderate windshears, the optimal trajectories are characterized by a monotonic climb; and (P2) for severe windshears, the optimal trajectories are characterized by an initial climb, followed by nearly horizontal flight, followed by renewed climbing after the aircraft has passed through the shear region.

In practice, an optimal trajectory is difficult to implement for two reasons: global information on the wind flowfield might not be available; even if it were available, there might not be enough computing capability onboard to process it adequately. However, optimal trajectories are important, because they can be used as benchmark trajectories for developing guidance laws as well as assessing the relative merits of guidance schemes.

Since global information on the wind flowfield is not available, one is forced to employ local information on the windshear and the downdraft. Under this assumption, guidance schemes were developed in Refs. 11-13 in order to approximate the behavior of the optimal trajectories and preserve properties (P1) and (P2). Noteworthy among the guidance schemes is the acceleration guidance scheme of Ref. 13.

In this paper, we present a gamma guidance scheme that approximates the behavior of the optimal trajectory and preserves properties (P1) and (P2). At first glance, the gamma guidance scheme and the acceleration guidance scheme are quite different. In the former, the control signal is the absolute path inclination; in the latter, the control signal is the relative acceleration. In implementation, the gamma guidance scheme and the acceleration guidance scheme yield almost identical results because the logical aspects of these guidance schemes are complementary to one another. The relation between gamma guidance and acceleration guidance is explored in this paper.

As stated above, the gamma guidance scheme requires local information on the windshear and the downdraft. Although this information will be available in future aircraft, it might not be available on current aircraft. This is why we present a simplified gamma guidance scheme (quick transition to horizontal flight) that is useful for flight in severe windshears. The simplified gamma guidance scheme yields trajectories that are close to the optimal trajectories in severe windshears; in addition, it is easy to implement as a practical piloting technique.

II. Equations of Motion

In this paper, we make use of the relative wind-axes system in connection with the following assumptions: 1) the aircraft is a particle of constant mass; 2) flight takes place in a vertical plane; 3) Newton's law is valid in an Earth-fixed system; 4) the wind flowfield is steady; and 5) maximum power setting is employed.

With the preceding premises, the equations of motion include the kinematical equations

$$\dot{x} = V\cos\gamma + W_x \tag{1a}$$

$$\dot{h} = V\sin\gamma + W_h \tag{1b}$$

and the dynamical equations

$$\dot{V} = (T/m)\cos(\alpha+\delta) - D/m - g\sin\gamma - (\dot{W}_x\cos\gamma + \dot{W}_h\sin\gamma) \tag{2a}$$

$$\dot{\gamma} = (T/mV)\sin(\alpha+\delta) + L/mV - (g/V)\cos\gamma + (1/V)(\dot{W}_x\sin\gamma - \dot{W}_h\cos\gamma) \tag{2b}$$

Because of assumption 4), the total derivatives of the wind velocity components and the corresponding partial derivatives satisfy the relations

$$\dot{W}_x = (\partial W_x/\partial x)(V\cos\gamma + W_x) + (\partial W_x/\partial h)(V\sin\gamma + W_h) \tag{3a}$$

$$\dot{W}_h = (\partial W_h/\partial x)(V\cos\gamma + W_x) + (\partial W_h/\partial h)(V\sin\gamma + W_h) \tag{3b}$$

These equations must be supplemented by the functional relations

$$T = T(h,V,\beta) \tag{4a}$$

$$D = D(h,V,\alpha), \qquad L = L(h,V,\alpha) \tag{4b}$$

$$W_x = W_x(x,h), \qquad W_h = W_h(x,h) \tag{4c}$$

and by the analytical relations

$$V_{ex} = V\cos\gamma + W_x, \qquad V_{eh} = V\sin\gamma + W_h \tag{5a}$$

$$V_e = \sqrt{(V_{ex}^2 + V_{eh}^2)}, \qquad \gamma_e = \arctan(V_{eh}/V_{ex}) \tag{5b}$$

$$\theta = \alpha + \gamma, \qquad \alpha_e = \alpha + \gamma - \gamma_e \tag{5c}$$

For a given value of the thrust inclination δ, the differential system (1-4) involves four state variables [the horizontal distance $x(t)$, the altitude $h(t)$, the velocity $V(t)$, and the relative path inclination $\gamma(t)$] and two control variables [the angle of attack $\alpha(t)$ and the power setting $\beta(t)$]. However, the number of control variables reduces to one (the angle of attack α) if the power setting β is specified in advance. The quantities defined by the analytical relations (5) can be computed a posteriori, once the values of x, h, V, γ, α, and β are known.

The angle of attack α and its time derivative $\dot{\alpha}$ are subject to the inequalities

$$\alpha \leq \alpha_* \tag{6a}$$

$$-\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_* \tag{6b}$$

where α_* is a prescribed upper bound and $\dot{\alpha}_*$ is a prescribed, positive constant.

III. Optimal Trajectories

In Ref. 13, optimal trajectories were computed by minimizing the modules of the maximum deviation of the absolute path inclination γ_e from the initial value γ_{e0}. Satisfaction of Eqs. (1-4) was enforced, together with satisfaction of inequalities (6). At the initial point, quasisteady flight was assumed; at the final point, relative path inclination recovery was assumed.

From the analysis of the optimal trajectories, the following basic properties were established in Ref. 13: (P1) for weak-to-moderate windshears, the optimal trajectories are characterized by a monotonic climb; and (P2) for severe windshears, the optimal trajectories are characterized by an initial climb, followed by nearly horizontal flight, followed by renewed climbing after the aircraft has passed through the shear region.

In addition, the following facts were also established:

F1) Initially, γ_e must be decreased until a certain critical value is reached. The critical value of γ_e is nearly maintained for a relatively long time interval. After passing through the shear region, the value of γ_e is gradually increased to $\gamma_e \cong \gamma_{e0}$.

F2) The critical value of γ_e depends on the intensity of the shear $\dot{W}_x/g$ and the intensity of the downdraft W_h/V; it decreases as the intensities of the shear and the downdraft increase.

IV. Gamma Guidance

In this section, we develop a gamma guidance scheme based on the absolute path inclination γ_e, whose objective is to approximate the behavior of the optimal trajectories in a windshear and, in particular, preserve properties (P1) and (P2).

From facts (F1) and (F2), we surmise that the gamma guidance law should have the form

$$\gamma_e = \tilde{\gamma}_e(\dot{W}_x/g,\ W_h/V) \tag{7a}$$

where

$$\tilde{\gamma}_e = C_1[1 - C_2(\dot{W}_x/g - W_h/V)], \qquad \tilde{\gamma}_{e1} \le \tilde{\gamma}_e \le \tilde{\gamma}_{e2} \tag{7b}$$

Here, C_1 and C_2 are suitable constants, to be derived from the study of the optimal trajectories; $\tilde{\gamma}_{e1}$ and $\tilde{\gamma}_{e2}$ are specified bounds for the absolute path inclination. More specifically, the constant C_1 is independent of the type of aircraft; its value is $C_1 = \gamma_{e0}$, the absolute path inclination at the initial point. The constant C_2 depends on the type of aircraft; its value is $C_2 \cong 4$ for a Boeing B-727 at sea level. The lower bound $\tilde{\gamma}_{e1}$ is either $\tilde{\gamma}_{e1} = 0$ or $\tilde{\gamma}_{e2} = \epsilon$, where ϵ is a small positive value. The upper bound $\tilde{\gamma}_{e2}$ is $\tilde{\gamma}_{e2} = \gamma_{e0}$, the absolute path inclination at the initial point.

Alternatively, providing γ_e can be measured, Eq. (7a) can be implemented through the feedback control law

$$\alpha - \tilde{\alpha}(V) = -K[\gamma_e - \tilde{\gamma}_e(\dot{W}_x/g, W_h/V)] \tag{8a}$$

$$\alpha \le \alpha_*, \qquad -\dot{\alpha}_* \le \dot{\alpha} \le \dot{\alpha}_* \tag{8b}$$

where K is the gain coefficient. In Eq. (8a), $\tilde{\alpha}(V)$ denotes the nominal angle of attack, whose structure is discussed in Sec. IV of Ref. 13; and $\tilde{\gamma}_e(\dot{W}_x/g, W_h/V)$ denotes the nominal absolute path inclination, which is supplied by Eq. (7b).

Starting from the absolute gamma guidance scheme, one can develop two additional guidance schemes: the relative gamma guidance scheme, based on the relative path inclination γ; and the theta guidance scheme, based on the pitch attitude angle θ. For the sake of brevity, the description of these alternative guidance schemes is omitted, and the reader is referred to Ref. 14 for details.

V. Relation of Gamma Guidance to Acceleration Guidance

Reference 13 presents an acceleration guidance scheme based on the relative acceleration. It is interesting to explore the relation of the gamma guidance scheme to the acceleration guidance scheme; indeed, in numerical computation, these schemes behave almost in the same way.

We consider the equations of motion (1) and (2). Upon combining Eqs. (1b) and (2a), we obtain the relation

$$\dot{V}/g + \dot{h}/V - (T/W)\cos(\alpha+\delta) + D/W + (\dot{W}_x/g)\cos\gamma + (\dot{W}_h/g)\sin\gamma - W_h/V = 0 \tag{9}$$

that can be rewritten in the form

$$\dot{V}/g + (\dot{h} - \dot{h}_*)/V + F = 0 \tag{10}$$

where

$$\dot{h}_*/V = (T/W)\cos(\alpha+\delta) - D/W \tag{11a}$$

$$F = (\dot{W}_x/g)\cos\gamma + (\dot{W}_h/g)\sin\gamma - W_h/V \tag{11b}$$

The interpretation of Eq. (10) is as follows: The first term is the relative acceleration, expressed in terms of the acceleration of gravity; the second term expresses the deviation of the rate of climb from its instantaneous quasisteady value in the absence of shear and downdraft; and the third term is the shear/downdraft factor, which combines the effects of the shear and the downdraft into a single entity.

If the simplifications

$$|\gamma_e| \ll 1, \qquad |\gamma| \ll 1, \qquad |W_x/V| \ll 1 \tag{12}$$

are employed in Eqs. (1) and (5), the approximate relations

$$\gamma_e = \dot{h}/V, \qquad \gamma_{e*} = \dot{h}_*/V \tag{13}$$

can be established. As a consequence, Eq. (10) becomes

$$\dot{V}/g + (\gamma_e - \gamma_{e*}) + F = 0 \tag{14}$$

If the further simplification

$$|\dot{W}_h\gamma/\dot{W}_x| \ll 1 \tag{15}$$

is employed, the shear/downdraft factor (11b) takes the form

$$F = \dot{W}_x/g - W_h/V \tag{16}$$

and Eq. (14) becomes

$$\dot{V}/g + (\gamma_e - \gamma_{e*}) + (\dot{W}_x/g - W_h/V) = 0 \tag{17}$$

The acceleration guidance law is based on the assumption of proportionality between the first term and the third term in Eq. (17), namely,

$$\dot{V}/g + C(\dot{W}_x/g - W_h/V) = 0 \tag{18a}$$

with the implication that

$$\gamma_e - \gamma_{e*} + (1-C)(\dot{W}_x/g - W_h/V) = 0 \tag{18b}$$

where C is a dimensionless constant. The following comments are pertinent:

1) Equation (18a) is the acceleration guidance law of Ref. 13; Eq. (18b) is an alternative form of the gamma guidance law of Sec. IV of this paper.

2) Equations (18a) and (18b) are consistent with the equation of motion, Eq. (17), for any value of the constant C.

3) Equations (18a) and (18b) are complementary to one another. If one views Eq. (18a) as the guidance law, then Eq. (18b) results from the equation of motion, Eq. (17); vice versa, if one views Eq. (18b) as the guidance law, then Eq. (18a) results from the equation of motion, Eq. (17).

VI. Simplified Gamma Guidance

The gamma guidance scheme of Sec. IV requires local information on the windshear and the downdraft. Although this information will be available in future aircraft, it might not be available on current aircraft. Therefore, we present a simplified gamma guidance scheme (quick transition to horizontal flight) that is useful for flight in severe windshears.

We recall that the essence of the gamma guidance law (8) is that the nominal absolute path inclination $\tilde{\gamma}_e$ decreases as the intensity of windshear/downdraft combination increases, tending to zero for severe windshear/downdraft combinations. Therefore, upon assuming that $\tilde{\gamma}_e = 0$, the gamma guidance law (8) simplifies to

$$\alpha - \tilde{\alpha}(V) = -K\gamma_e \tag{19a}$$

$$\alpha \le \alpha_*, \qquad -\dot{\alpha}_* \le \dot{\alpha} \le \dot{\alpha}_* \tag{19b}$$

Because it might be difficult to measure γ_e, we convert the simplified gamma guidance scheme (19) into a rate-of-climb format that is more suitable for in-flight measurements. Upon combining Eqs. (1) and (5), and using the assumptions (12), we see that the absolute path inclination can be rewritten as

$$\gamma_e = \dot{h}/V \tag{20}$$

As a consequence, the simplified gamma guidance law (19) becomes

$$\alpha - \tilde{\alpha}(V) = -K(\dot{h}/V) \tag{21a}$$

$$\alpha \le \alpha_*, \qquad -\dot{\alpha}_* \le \dot{\alpha} \le \dot{\alpha}_* \tag{21b}$$

In a windshear encounter, the relative change of V is small by comparison with the relative change of $\dot{h}$. Therefore, upon replacing V with its average value $\bar{V}$ and upon redefining the gain coefficient as follows:

$$\bar{K} = K/\bar{V} \tag{22}$$

relations (21) become

$$\alpha - \tilde{\alpha}(V) = -\bar{K}\,\dot{h} \tag{23a}$$

$$\alpha \leq \alpha_*, \qquad -\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_* \tag{23b}$$

Then, upon dropping the bar, relations (23) are formally rewritten as

$$\alpha - \tilde{\alpha}(V) = -K\,(\dot{h} - 0) \tag{24a}$$

$$\alpha \leq \alpha_*, \qquad -\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_* \tag{24b}$$

The simplified gamma guidance law represented by relations (24) yields a quick transition to horizontal flight if the gain coefficient K is chosen properly. This law is particularly suitable for flight in a severe windshear. Indeed, the essence of safe flight in a severe windshear is to try to avoid the altitude loss, while simultaneously containing the velocity loss. This conception, embodied in the simplified gamma guidance scheme (quick transition to horizontal flight), goes against the intuitive belief of some people that a crash can be avoided by climbing as high as possible via maximum angle of attack. If maximum angle of attack is employed, the initial gain of altitude is coupled with severe velocity loss and then is followed by considerable loss of altitude, occasionally resulting in a crash.[15]

The simplified gamma guidance law (24) is to be employed in the windshear portion of the trajectory. In the aftershear portion, relations (24) are to be followed by

$$\alpha - \tilde{\alpha}(V) = -K\,(\dot{h} - \dot{h}_R) \tag{25a}$$

$$\alpha \leq \alpha_*, \qquad -\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_* \tag{25b}$$

where $\dot{h}_R$ denotes a constant reference value, for instance, $\dot{h}_R = \dot{h}_0$.

VII. Data for the Examples

In this section, we summarize the main data used in the numerical experiments.

Aircraft

The aircraft under consideration is a Boeing B-727 aircraft powered by three JT8D-17 turbofan engines.[9] It is assumed that the aircraft has become airborne from a runway located at sea level; the ambient temperature is 100°F; the gear is up; the flap setting is $\delta_F = 15$ deg; the engines are operating at maximum power setting; and the takeoff weight is $W = 180{,}000$ lb.

Windshear Model

The windshear model is decribed by the following relations, valid for $h \leq 1000$ ft:[16,17]

$$W_x = \lambda A(x) \tag{26a}$$

$$W_h = \lambda (h/h_*) B(x) \tag{26b}$$

with the implication that

$$\Delta W_x = 100\lambda \tag{26c}$$

$$\Delta W_h = 50\lambda (h/h_*) \tag{26d}$$

Here, the parameter λ characterizes the intensity of the windshear/downdraft combination; the function $A(x)$ represents the distribution of the horizontal wind vs the horizontal distance (Fig. 1); the function $B(x)$ represents the distribution of the vertical wind vs the horizontal distance (Fig. 1); and h_* is a reference altitude, $h_* = 1000$ ft. Also, ΔW_x is the horizontal wind velocity difference (maximum tailwind minus maximum headwind) and ΔW_h is the vertical wind velocity difference (maximum updraft minus maximum downdraft).

Concerning the horizontal wind, Eq. (26a), the function $A(x)$ represents a linear transition from a uniform headwind of -50 ft s^{-1} to a uniform tailwind of $+50$ ft s^{-1}. Hence, the wind velocity difference is $\Delta W_x = 100$ ft s^{-1} if $\lambda = 1$. The transition takes place over a distance $\Delta x = 4600$ ft, starting at $x = 0$ ft and ending at $x = 4600$ ft; the average wind gradient over the horizontal distance interval $300 \leq x \leq 4300$ ft is $\Delta W_x / \Delta x \cong 0.025$ s^{-1} if $\lambda = 1$.

Concerning the vertical wind, Eq. (26b), the function $B(x)$ has a bell-shaped form; in particular, the downdraft vanishes at $x = 0$ and 4600 ft and achieves maximum negative value at $x = 2300$ ft. This maximum negative value is -50 ft s^{-1} if $h = 1000$ ft and $\lambda = 1$; hence, $\Delta W_h = 50$ ft s^{-1} if $h = 1000$ ft and $\lambda = 1$.

To sum up, the windshear model (26) has the following properties: 1) it represents the transition from a headwind to a tailwind, with nearly constant shear in the core of the downburst; 2) the downdraft achieves maximum negative value at the center of the downburst; 3) the downdraft vanishes at $h = 0$; and 4) the functions W_x, W_h nearly satisfy the continuity equation and the irrotationality condition in the core of the downburst.

Inequality Constraints

The angle of attack and its time derivative are subject to inequalities (6), with

$$\alpha_* = 16 \text{ deg}, \qquad \dot{\alpha}_* = 3 \text{ deg s}^{-1} \tag{27}$$

Initial Conditions

The following initial conditions are assumed:

$$x(0) = 0 \text{ ft} \tag{28a}$$

$$h(0) = 50 \text{ ft} \tag{28b}$$

$$V(0) = 276.8 \text{ ft s}^{-1} \tag{28c}$$

$$\gamma(0) = 6.989 \text{ deg} = 0.1220 \text{ rad} \tag{28d}$$

$$\alpha(0) = 10.36 \text{ deg} = 0.1808 \text{ rad} \tag{28e}$$

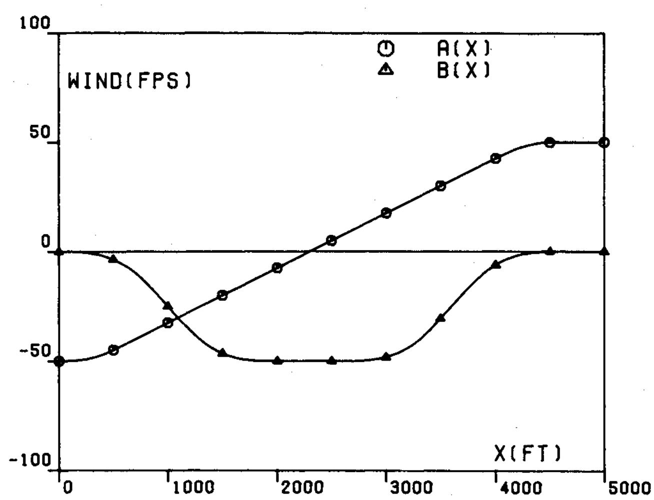

Fig. 1 Wind functions $A(x)$, $B(x)$.

We note that the values (28) correspond to quasisteady flight; also, the initial velocity $V(0)$ is Federal Aviation Administration certification velocity V_2, augmented by 10 knots; in turn, the velocity V_2+10 (in knots) corresponds approximately to the steepest climb condition in quasisteady flight.

Final Conditions

The final time is set at the value

$$\tau = 40 \text{ s} \tag{29}$$

This is about twice the duration of the windshear encounter ($\Delta t = 18$ s).

VIII. Numerical Results, Gamma Guidance

Using the data of Sec. VII, numerical results were obtained for the gamma guidance scheme of Sec. IV. This scheme was implemented in the feedback control form (8) in conjunction with the following constants:

$$C_1 = \gamma_{e0} \tag{30a}$$

$$C_2 = 4 \tag{30b}$$

$$\tilde{\gamma}_{e1} = 0 \tag{30c}$$

$$\tilde{\gamma}_{e2} = \gamma_{e0} \tag{30d}$$

$$K = 5 \tag{30e}$$

The value of γ_{e0} was computed with Eqs. (5), based on the initial conditions (28) and the prescribed wind intensity parameter λ [see Eqs. (26)].

Four values of the wind intensity parameter were considered, namely, $\lambda = 0.6$, 0.8, 1.0, 1.1; the corresponding wind velocity difference ΔW_x [see Eq. (26c)] and initial absolute path inclination [see Eqs. (5)] are shown in Table 1. For these values, both optimal trajectories (OT) and gamma guidance trajectories (GGT) were computed. The associated altitude distribution $h(t)$ is shown in Fig. 2, which includes four parts: Fig. 2a refers to $\Delta W_x = 60$ ft s^{-1}; Fig. 2b to $\Delta W_x = 80$ ft s^{-1}; Fig. 2c to $\Delta W_x = 100$ ft s^{-1}; and Fig. 2d to $\Delta W_x = 110$ ft s^{-1}.

Figure 2 shows that there is remarkable qualitative agreement between the GGT and the OT; indeed, the GGT

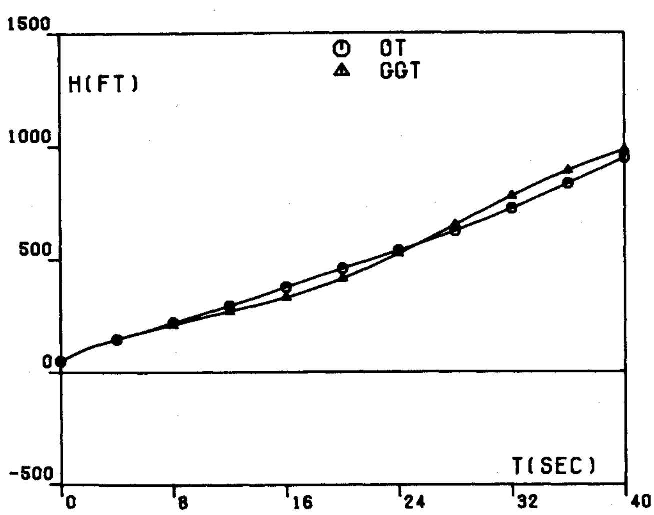

Fig. 2a Comparison of GGT and OT for $\Delta W_x = 60$ ft s^{-1}: altitude h vs time t.

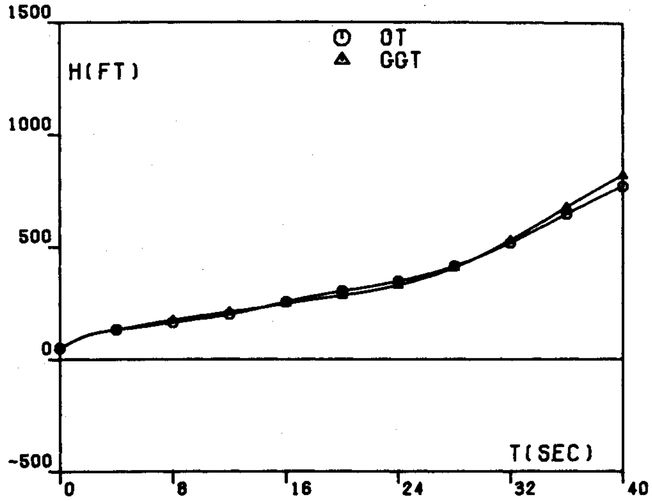

Fig. 2b Comparison of GGT and OT for $\Delta W_x = 80$ ft s^{-1}: altitude h vs time t.

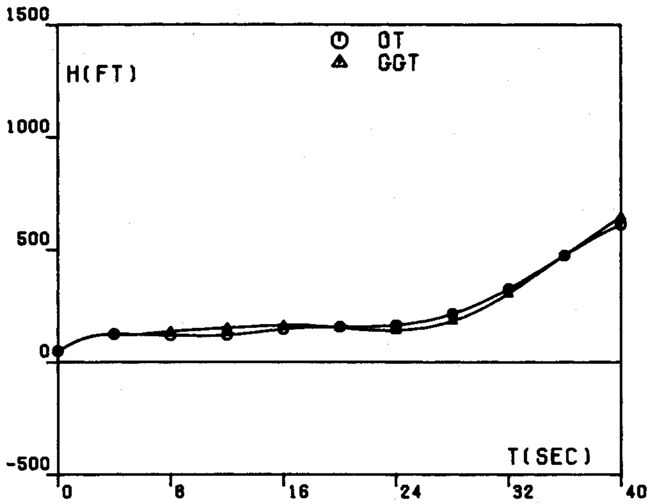

Fig. 2c Comparison of GGT and OT for $\Delta W_x = 100$ ft s^{-1}: altitude h vs time t.

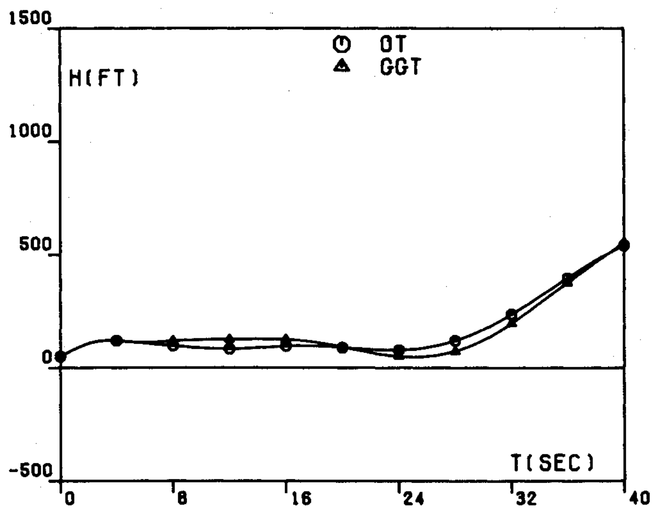

Fig. 2d Comparison of GGT and OT for $\Delta W_x = 110$ ft s^{-1}: altitude h vs time t.

Table 1 Absolute path inclination at the initial point

λ	ΔW_x, ft s^{-1}	γ_{e0}, deg	γ_{e0}, rad
0.6	60	7.84	0.1368
0.8	80	8.16	0.1424
1.0	100	8.52	0.1487
1.1	110	8.71	0.1520

preserves the basic properties of the OT, specifically: (P1) for weak-to-moderate windshears, the GGT is characterized by a monotonic climb; and (P2) for strong-to-severe windshears, the GGT is characterized by an initial climb, followed by nearly horizontal flight, followed by renewed climbing after the aircraft has passed through the shear region.

IX. Numerical Results, Simplified Gamma Guidance

Using the data of Sec. VII, numerical results were obtained for the simplified gamma guidance scheme of Sec. VI. This scheme was implemented in the feedback control form (24) and (25) in conjunction with the following constants:

$$K = 0.004 \text{ rad ft}^{-1}\text{ s} \tag{31a}$$

$$\dot{h}_R = \dot{h}_0 = 33.68 \text{ ft s}^{-1} \tag{31b}$$

Two values of the wind intensity parameter were considered, namely, $\lambda = 1.0$, 1.1. The corresponding wind velocity differences are $\Delta W_x = 100$, 110 ft s^{-1}. For these values, both the OT and the SGGT were computed. The associated altitude distribution $h(t)$ is shown in Fig. 3, which includes two parts: Fig. 3a refers to $\Delta W_x = 100$ ft s^{-1}; and Fig. 3b to $\Delta W_x = 110$ ft s^{-1}.

Figure 3 shows that there is qualitative agreement between the SGGT and the OT, but that this agreement is better in the shear portion of the trajectory ($t \leq 18$ s) than in the aftershear portion of the trajectory ($t \geq 18$ s). In the aftershear portion, the OT is characterized by higher altitudes than the SGGT, and the difference of altitude increases as the wind velocity difference increases. With reference to the aftershear guidance law (25), one can easily improve performance and generate a better agreement with the OT by replacing the reference rate of climb (31b) with a somewhat larger value.

For $\lambda = 1.0$, corresponding to $\Delta W_x = 100$ ft s^{-1}, the SGGT was compared with the constant pitch trajectory (CPT, $\theta = 17.35$ deg) and the maximum angle of attack trajectory (MAAT, $\alpha = 16.00$ deg). In computing the CPT, it was postulated that the target pitch is identical with the initial pitch. In computing the MAAT, it was postulated that the transition from the initial angle of attack to the maximum angle of attack is performed at the maximum permissible rate ($\dot{\alpha} = 3$ deg s^{-1}).

The comparison between SGGT, CPT, and MAAT is shown in Fig. 4, which includes four parts: Fig. 4a refers to the altitude distribution $h(t)$; Fig. 4b to the velocity distribution $V(t)$; Fig. 4c to the angle of attack distribution $\alpha(t)$; and Fig. 4d to the pitch distribution $\theta(t)$. The following points must be noted.

Altitude

The SGGT is characterized by an initial climb, followed by nearly horizontal flight, followed by renewed climbing after the aircraft has passed through the shear region. For the CPT, the function $h(t)$ is wavelike in the following sense: the aircraft climbs initially, then loses altitude, then barely avoids the ground, and finally climbs again. For the MAAT, the function $h(t)$ is also wavelike, but with wider amplitude of the altitude oscillations: the aircraft climbs initially to considerable altitude, then loses altitude quickly, and finally hits the ground.

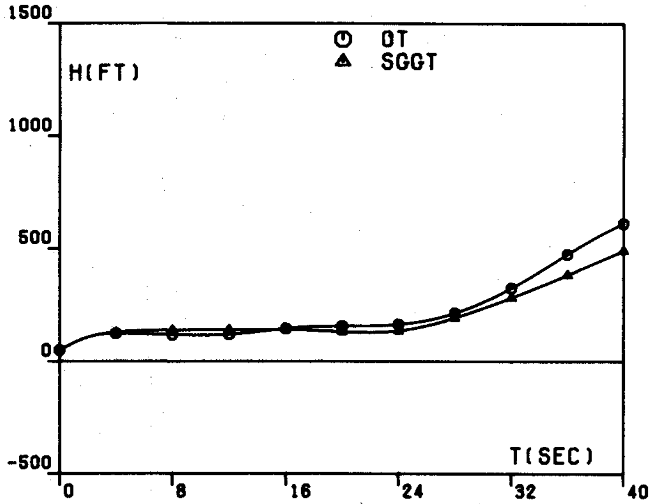

Fig. 3a Comparison of SGGT and OT for $\Delta W_x = 100$ ft s^{-1}: altitude h vs time t.

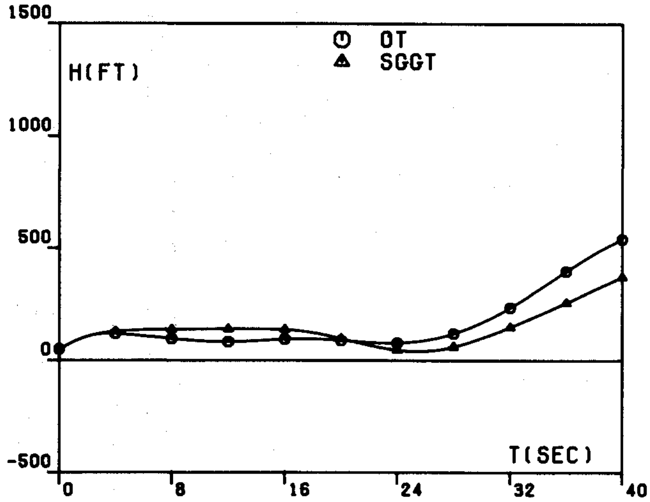

Fig. 3b Comparison of SGGT and OT for $\Delta W_x = 110$ ft s^{-1}: altitude h vs time t.

Velocity

The function $V(t)$ of the SGGT is characterized by decreasing velocity in the shear region and increasing velocity in the aftershear region; the point of minimum velocity is reached at the end of the shear ($t = 18$ s). For the CPT, the point of minimum velocity is also reached at the end of the shear ($t = 18$ s); however, the velocity drop due to windshear action is larger in the CPT than in the SGGT. For the MAAT, the point of minimum velocity is reached before the end of the shear ($t = 14$ s); also, the drop in velocity due to windshear action is much larger in the MAAT than in the SGGT. These characteristics of the MAAT are undesirable: the sharp increase in velocity in the shear region implies a severe loss of altitude, resulting in a crash.

Angle of Attack

The function $\alpha(t)$ of the SGGT is close to the function $\alpha(t)$ of the OT: the angle of attack of the SGGT exhibits an initial decrease, followed by a gradual, sustained increase until the angle of attack boundary is reached near the end of the shear; the initial decrease in the angle of attack is important to performance in a windshear and engenders the quick transition to horizontal flight that characterizes the SGGT. For the CPT, the function $\alpha(t)$ is nearly constant at the beginning of the shear and increases afterward. For the MAAT, the function $\alpha(t)$ is constant at all times, except in the time interval $0 \le t \le \sigma$, where $\sigma = (\alpha_* - \alpha_0)/\dot{\alpha}_0$; in this interval, the transition from the initial angle of attack α_0 to the stick-shaker angle of attack α_* takes place at the constant rate $\dot{\alpha} = \dot{\alpha}_0$.

Pitch Attitude

The function $\theta(t)$ of the SGGT is close to the function $\theta(t)$ of the OT: the pitch attitude of the SGGT exhibits an initial decrease, followed by a gradual, mild increase. For the CPT, the function $\theta(t)$ is constant at all times, except for the time interval $14 \le t \le 26$ s, which encompasses the end of the shear ($t = 18$ s). In this time interval, lower values of the pitch are required, owing to the fact that the angle of attack boundary has been reached. For the MAAT, the function $\theta(t)$ has a wave-like behavior, with wide amplitude oscillations ($\Delta\theta \cong 50$ deg).

X. Survival Capability

In this section, we analyze the survival capability of an aircraft in a severe windshear. Indicative of this survival capability is the windshear/downdraft combination that results in the minimum altitude being equal to the ground altitude.

To analyze this important problem, we consider the one-parameter family of windshear models (26). Here, the parameter λ characterizes the intensity of the windshear/downdraft combination. By increasing the value of λ, more intense windshear/downdraft combinations are generated until a critical value λ_c is found such that $h_{min} = 0$ for a particular guidance scheme.

Within this framework, we assess the survival capability of the following five trajectories:

1) The optimal trajectory (OT). This is the trajectory minimaximizing the modulus of the difference $\Delta\gamma_e$ between the absolute path inclination γ_e and the initial value γ_{e0}.

2) The gamma guidance trajectory (GGT, Secs. IV and VIII), with a gain coefficient $K = 5$.

3) The simplified gamma guidance trajectory (SGGT, Secs. VI and IX), with three values of the gain coefficient, namely, $K = 0.004$, 0.008, 0.020 rad ft^{-1} s.

4) The constant pitch trajectory (CPT, Sec. IX), with $\theta = 17.35$ deg (see Ref. 18).

5) The maximum angle of attack trajectory (MAAT, Sec. IX), with $\alpha = 16.00$ deg.

For the preceding guidance schemes, the numerical results are given in Table 2, which contains the following information: the initial altitude h_0; the maximum altitude h_{max} (first relative maximum); the minimum altitude $h_{min} = 0$; the critical value of the intensity parameter λ_c; the critical value of the wind velocity difference ΔW_{xc}; and the efficiency ratio R,

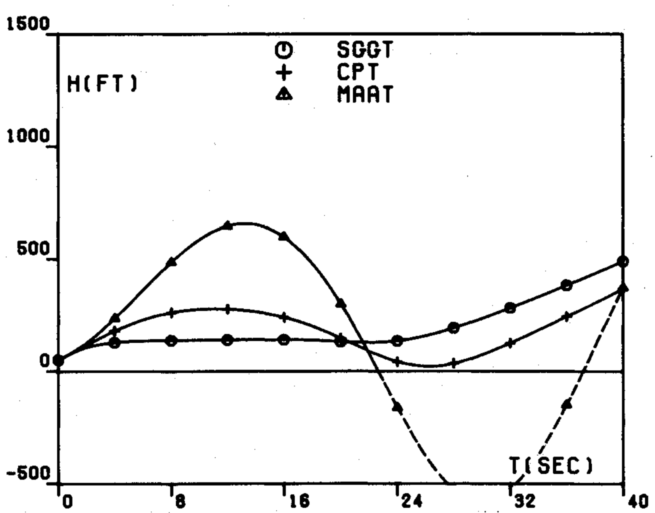

Fig. 4a Comparison of SGGT, CPT, and MAAT for $\Delta W_x = 100$ ft s^{-1}: altitude h vs time t.

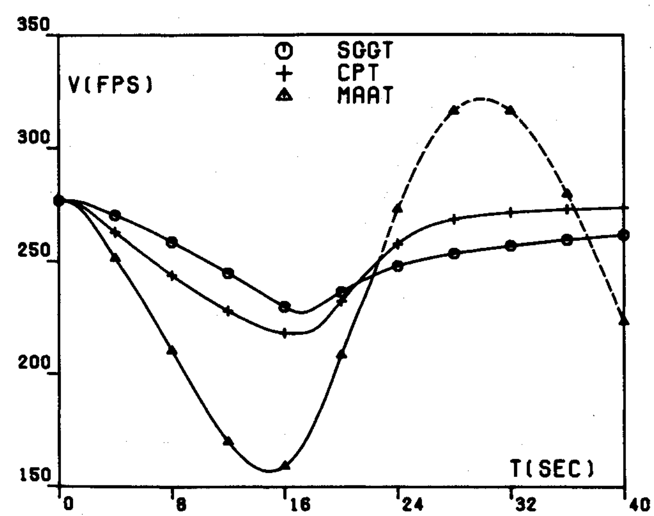

Fig. 4b Comparison of SGGT, CPT, and MAAT for $\Delta W_x = 100$ ft s^{-1}: relative velocity V vs time t.

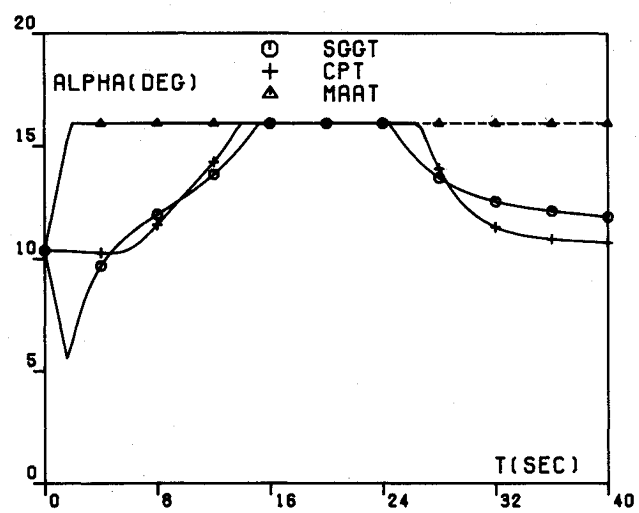

Fig. 4c Comparison of SGGT, CPT, and MAAT for $\Delta W_x = 100$ ft s^{-1}: relative angle of attack α vs time t.

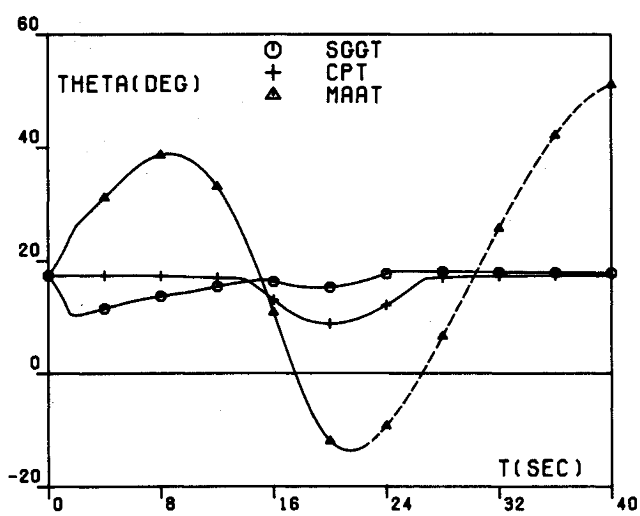

Fig. 4d Comparison of SGGT, CPT, and MAAT for $\Delta W_x = 100$ ft s^{-1}: pitch attitude angle θ vs time t.

Table 2 Survival capability of different trajectories

Trajectory	Remark	h_0, ft	h_{max}, ft	h_{min}, ft	λ_c	ΔW_{xc}, ft s^{-1}	R
OT	minmax $\lvert\Delta\gamma_e\rvert$	50	118.5	0	1.195	119.5	1.000
GGT[a]	$K=5$	50	119.8	0	1.152	115.2	0.964
SGGT[b]	$K=0.004$	50	140.8	0	1.134	113.4	0.949
SGGT[b]	$K=0.008$	50	116.9	0	1.162	116.2	0.972
SGGT[b]	$K=0.020$	50	115.0	0	1.194	119.4	0.999
CPT[c]	$\theta=17.35$	50	277.0	0	1.017	101.7	0.851
MAAT[d]	$\alpha=16.00$	50	765.4	0	0.576	57.6	0.482

[a]Value of K is dimensionless; [b]Value of K is in rad ft^{-1} s; [c]Value of θ is in deg; [d]Value of α is in deg.

defined to be

$$R=(\lambda_c)_{PT}/(\lambda_c)_{OT}=(\Delta W_{xc})_{PT}/(\Delta W_{xc})_{OT} \tag{32}$$

Here, the subscript PT denotes a particular trajectory and the subscript OT denotes the optimal trajectory. From Table 2, the following conclusions can be derived:

1) The survival capability of the SGGT is close to the survival capability of the OT and the GGT.

2) The survival capability of the SGGT is superior to that of the CPT and is much superior to that of the MAAT.

3) The survival capability of the SGGT improves slightly as the gain coefficient K increases; larger values of K correspond to more energetic transitions to horizontal flight.

XI. Conclusions

This paper is concerned with guidance schemes for near-optimum performance in a windshear. The takeoff problem is considered with reference to flight in a vertical plane. In addition to the horizontal shear, the presence of a downdraft is assumed.

A gamma guidance scheme, based on the absolute path inclination, is presented. In this approach, local information on the windshear and the downdraft is needed. Numerical experiments show that the gamma guidance scheme produces trajectories that preserve the basic properties of the optimal trajectories, specifically: (P1) for weak-to-moderate windshears, the optimal trajectories are characterized by a monotonic climb; and (P2) for severe windshears, the optimal trajectories are characterized by an initial climb, followed by nearly horizontal flight, followed by renewed climbing after the aircraft has passed through the shear region.

As previously stated, the gamma guidance scheme requires local information on the windshear and the downdraft. Although this information will be available in future aircraft, it might not be available on current aircraft. Hence, we present a simplified gamma guidance scheme, useful for flight in severe windshears.

For relatively severe windshears, the simplified gamma guidance scheme yields trajectories that are competitive with those produced by the gamma guidance scheme. In addition, for relatively severe windshears, the simplified gamma guidance trajectory has a better survival capability than the constant pitch trajectory and the maximum angle of attack trajectory.

The simplified gamma guidance scheme is simple in concept as well as in flight implementation, since it does not require the precise knowledge of the windshear. It results in a piloting strategy called quick transition to horizontal flight. More specifically, upon sensing that the aircraft is in a shear, the pilot performs a quick transition from climbing flight to horizontal flight, and then keeps the plane in nearly horizontal flight until the shear region is past.

Acknowledgments

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